Games, graphs, and machines

Modular arithmetic

August 2, 2024

A cautionary tale

On $\mathbb{R}$, say $a \sim b$ if $a - b \in \mathbb{Z}$.

Is this an equivalence relation?

Reflexive ✓

Symmetric ✓

Transitive ✓

A cautionary tale

On $\mathbb{R}$, say $a \sim b$ if $a - b \in \mathbb{Z}$.

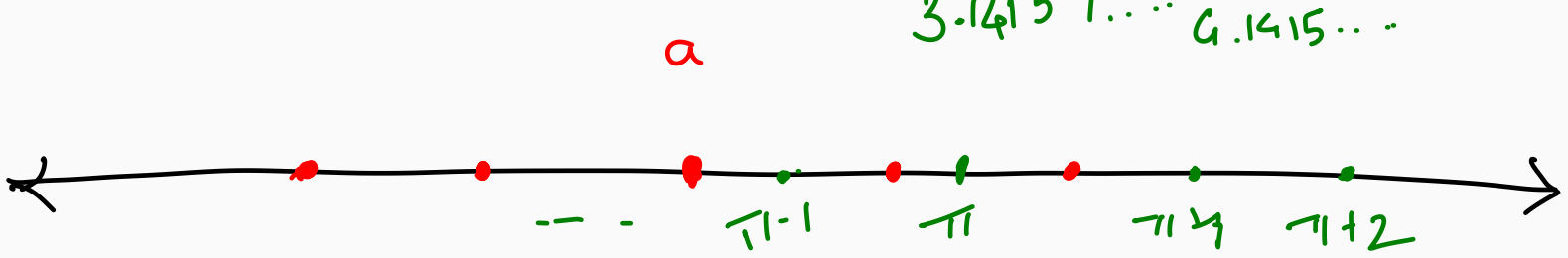
Is this an equivalence relation?

What are the equivalence classes?

what's $[1]$ $[\pi]$ $[1.1]$ $[3] = \mathbb{Z}$

$$[a] = \{a, a+1, a+2, \ldots a-1, a-2 \ldots\}$$

$$= \{a + k \mid k \in \mathbb{Z}\}$$



A cautionary tale

Let $\overline{R}$ be the set of equivalence classes. Define $+$ on equivalence classes by the rule

$$[a] + [b] = [a + b].$$

Is this well-defined?

What do we need to check?

Consistency: choosing different reps on LHS gives same eqv class on RHS.

$\in$ $\in$

$(a+m)$ $(b+n)$

$[a+b+m+n] = [a+b]$.

A cautionary tale

Let $\overline{R}$ be the equivalence classes. Define $\times$ on equivalence classes by the rule

$$[a] \times [b] = [a \times b].$$

NO

Is this well-defined?

Do a consistency check.

$[1.5] \times [1.1]$

1.65

$[2.5] \times [1.1]$

2.75

Instead of a, pick $a+m$

b $\quad$ $b+n$

ab $\quad$ $ab + an + bm + mn$

Linear equations

Find all $x \in \mathbb{Z}/5\mathbb{Z}$ such that

$$\overline{2} \cdot x + \overline{7} = 0.$$

More equations

Find all $x \in \mathbb{Z}/8\mathbb{Z}$ such that

$$\overline{x}^2 = 1.$$

Exponentiation

What is 2^{2024} modulo 7?